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Proof by G. E. WAHLIN, Urbana, Ill.

$|1 - \sqrt{3}| < 1$ . Hence (1)  $0 < (1 - \sqrt{3})^{2n} < 1$ .

But  $(1 + \sqrt{3})^{2n} + (1 - \sqrt{3})^{2n} = E$ , a rational integer, and since (1) is true, this is the integer next above  $(1 + \sqrt{3})^{2n}$ .

Since  $\frac{(1 \pm \sqrt{3})^2}{2} = 2 \pm \sqrt{3}$ , we have  $\frac{(1 \pm \sqrt{3})^{2n}}{2^n} = (2 \pm \sqrt{3})^n = P \pm Q\sqrt{3}$ ,  $P$  and  $Q$  rational integers.

Then  $\frac{(1 + \sqrt{3})^{2n}}{2^n} + \frac{(1 - \sqrt{3})^{2n}}{2^n} = \frac{E}{2^n} = 2P$ . Therefore,  $\frac{E}{2^{n+1}} = P$ , and hence  $E$  is divisible by  $2^{n+1}$ .

Also solved by J. Scheffer, G. B. M. Zerr, and V. M. Spunar.

## PROBLEMS FOR SOLUTION.

### ALGEBRA.

321. Proposed by C. C. BLAND, Attorney at Law, Rolla, Mo.

A corporation is capitalized for \$20,000. 125 shares of the par value of \$100 per share has been issued. A has 27  $\frac{19}{78}$  shares. B, C, D, E and F each have 19  $\frac{43}{78}$  shares. It is the wish of the corporation to cancel the certificates held by A, B, C, D, E, and F, and to issue new certificates to each of them in lieu of those now held by them, and to avoid the issuance of any certificate for a fraction of a share. How many shares should each receive, the whole not to exceed 200, at the same time maintaining the present interest of each in the corporation?

322. Proposed by THEODORE L. DeLAND, Treasury Department, Washington, D. C.

Take six consecutive prime numbers, as 53, 59, 61, 67, 71, and 73, and find the least whole number such that if it be divided by 59 the remainder will be 53, if it be divided by 67 the remainder will be 61, and if it be divided by 73 the remainder will be 71, and show that this least whole number and the succeeding consecutive whole numbers that will fulfill this condition as to divisions and remainders are in arithmetical progression; and also show whether or not this is a general law for  $n$  consecutive prime numbers; and if there be such a general law whether or not that general law will lead to a general law for the finding of prime numbers.

323. Proposed by E. B. ESCOTT, Ann Arbor, Mich.

Show that the relation  $(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2$  can hold for real numbers only when they are in proportion.

### GEOMETRY.

348. Proposed by W. J. GREENSTREET, M. A., Marling School, Stroud, England.

Two parabolas and a rectangular hyperbola circumscribe a given quadrilateral. Find a relation between the squares of the latera recta of the parabolas and the squares of the perpendiculars from the center of the hyperbola to the axes of the parabolas.

349. Proposed by J. A. CAPARO, Notre Dame University, Notre Dame, Indiana.

Given the radius of a circular smooth cylinder and its position with respect to a source of light and the eye. Find a geometrical construction to determine the line of brilliancy.

350. Proposed by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Given the quadrilateral  $AB=a=225$ ,  $BC=b=153$ ,  $CD=c=207$ ,  $DA=d=135$ ,  $AC=e=240$ . Find the side of the square inscribed in this quadrilateral having a corner in each side.

## CALCULUS.

279. Proposed by L. H. McDONALD, M. A., Ph. D., Sometime Tutor at Cambridge, Jersey City, N. J.

Find the ellipse of minimum area which will pass through the vertices of a triangle. (Hedrick-Goursat's *Math. Anal.*, p. 133, ex. 9.)

280. Proposed by C. N. SCHMALL, 89 Columbia Street, New York.

Find the envelope of the system of spheres  

$$\left. \begin{aligned} (x-a)^2 + (y-b)^2 + z^2 &= r^2 \\ a^2 + b^2 &= c^2 \end{aligned} \right\}.$$

281. Proposed by S. A. COREY, Hiteman, Iowa.

Prove that if  $n$  be a positive integer greater than unity,

$$\text{Log} n = (1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n-1} + \frac{1}{2n}) - C + \frac{B_1}{2n^2} - \frac{B_2}{4n^4} + \frac{B_3}{6n^6} - \frac{B_4}{8n^8} + \dots (1).$$

REMARK.—This development may be obtained by employing the formula given by the proposer in *Annals of Mathematics*, second series, Vol. 5, No. 4, July, 1904, but other proofs are also desired. It will be observed that (1) offers a ready method of evaluating  $C$ , which is remarkably simple and very rapidly convergent if  $n > \text{or} = 10$ . Compare with method given by Mr. Bromwich in *Messenger of Mathematics*, October, 1906.

## MECHANICS.

233. Proposed by W. J. GREENSTREET, M. A., Marling School, Stroud, England.

A shot, mass  $m$ , is fired from a gun, mass  $M$ , which can move on a horizontal plane. The muzzle velocity of the shot is given, the muzzle always points in the same direction as the shot leaves it. The envelope of the trajectories is a parabola. Find the ratio of the distances of its focus and directrix from the plane in terms of  $M$  and  $m$ .

234. Proposed by C. N. SCHMALL, 89 Columbia Street, New York City.

A ladder is placed with one end resting against a smooth wall and making with it an angle  $\phi$ . Also, the roughness of the ground prevents it from slipping. A man weighing as much as the ladder ascends to the top. Taking  $\mu$  as the coefficient of friction, prove:

(a) The ladder will slip before he gets to the top if  $\phi > \tan^{-1} 4\mu/3$ .

(b) If the ascent be feasible, there will be three times as much friction when he is at the top as when he is at the bottom, (See Jeans' *Theoretical Mechanics*, p. 47.)

235. Proposed by W. J. GREENSTREET, M. A., Marling School, Stroud, England.

A uniform heavy rod turns freely round a hinge at one end and rests with the other against a rough vertical wall, at angle,  $\alpha$ , to the wall. Find the angle of arc on which this end may rest, and the pressures at the ends of the arc.